# COLLECTORS IN A PLASMA

# (or Probes and Sheaths)

## Contents

## Trapping Probes

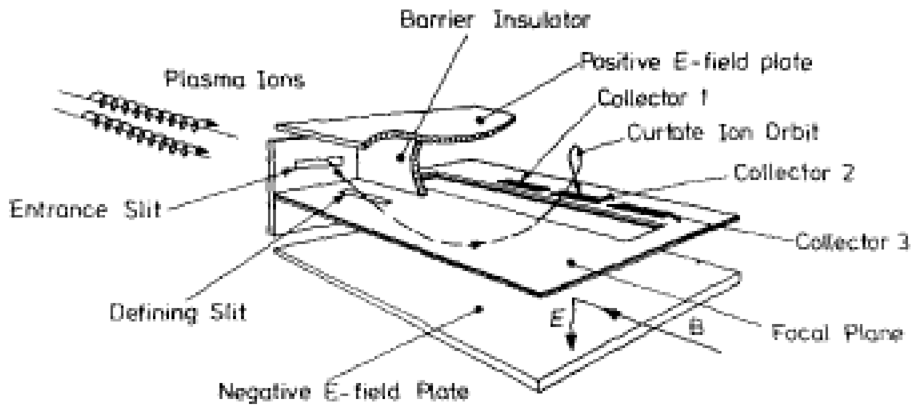



*Figure 14. Schematic of the DITE plasma ion mass spectrometer.*

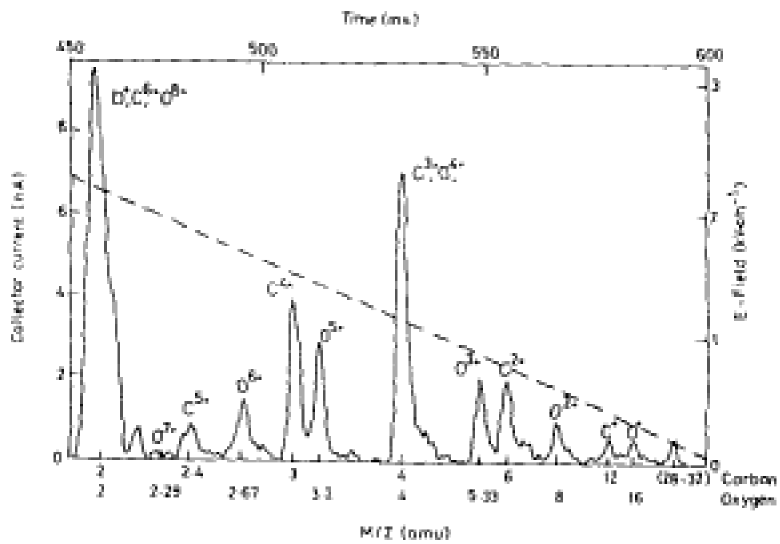



*Figure 15. Mass spectrum in the edge of a deuterium plasma in DITE.*

The mixture of charge states in an edge plasma can be measured with a plasma ion spectrometer. The instrument utilizes a strong B field (that used to confine the plasma) and a perpendicular E field within the analyzer to separate ions according to the ratio of charge to mass. Typical spectra are obtained by ramping the analyzer electric field to sweep the ion peaks across an array of three detectors.

# Gridded analyzer

Measures electron and ion energy distribution functions in a plasma where T < 50 eV. It is perturbing, because of its size.

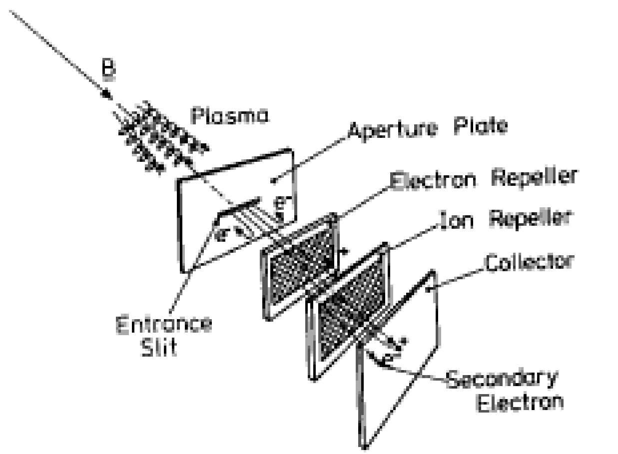



*Figure 15. Schematic of the DITE retarding field analyzer.*

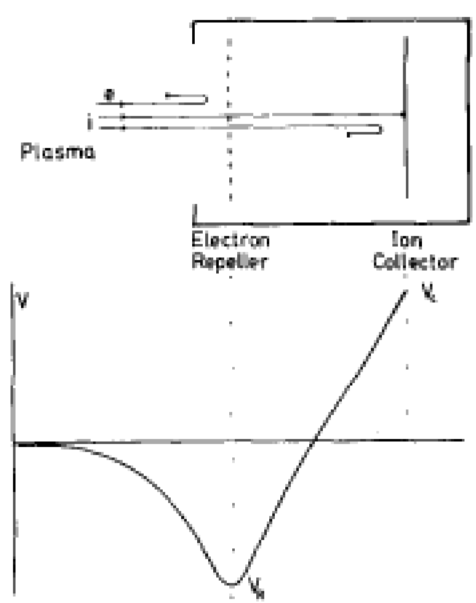



Figure showing simple gridded analyzer.

Plasma particles approach a collector after passing through a grid. The grid is biased to repel either electrons or ions. When the collector voltage $V_C$ is varied, only (e.g.) ions with E >

$eV_c$ will be collected. Then the logarithmic slope of the collection current against voltage for $V_c > 0$ gives the ion temperature. A problem is secondary electron emmission caused by ions or electrons hitting electrode surfaces. When collector is repelling all but a few percent of ions, electron current from secondary electrons is a problem. This is overcome using a more elaborate grid system. First grid repels plasma electrons. Second grid is an ion repeller whose potential is varied. Third grid is the electron suppresser. Another difficulty is space charge limitation. Then potential applied is not that which really exists. Worst case between electron and ion repellers, when bulk of ions cab just reach the repeller. To avoid the problem should set grid separation < Debye length - not possible at high density.

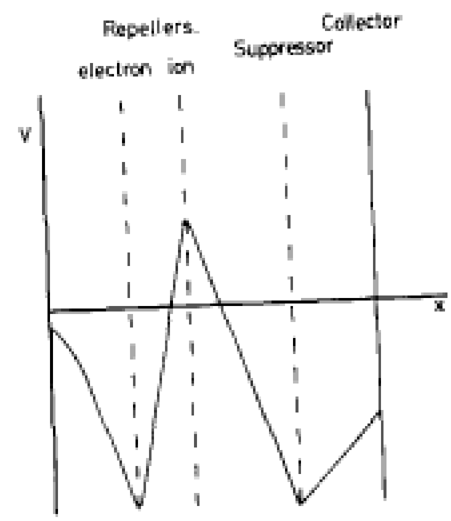
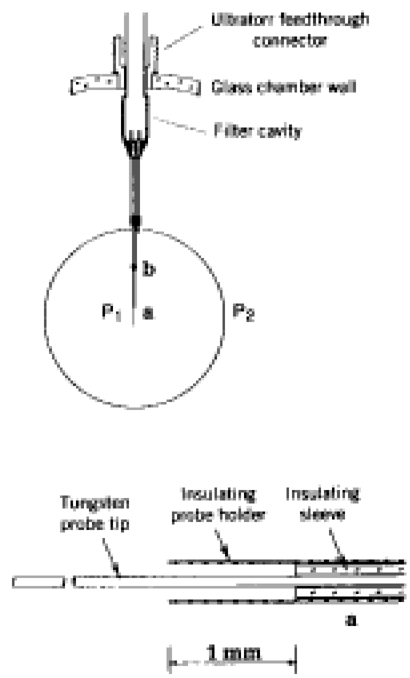


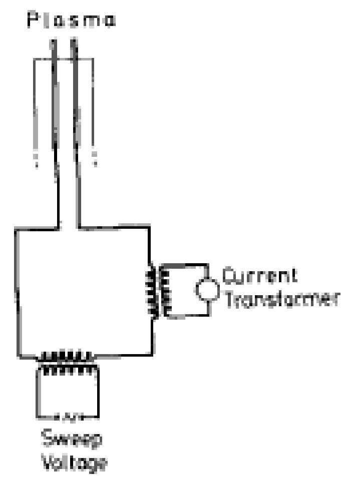

Figure showing more complex grid configuration.

## Emissive probes

Measures plasma or space potential (see later). This is not the potential at which the probe would 'float' at, because a conductor in a plasma must charge up negative to repel electrons and ensure an ambipolar collection of particles.




A typical probe for use in a plasma.




A really bad figure of how you might measure the 'characteristics'

A simple probe is heated to emit electrons all the time (thermionic). If the probe potential is positive wrt the plasma then electrons emitted with low energy cannot escape and are attracted back. The probe current is therefore unchanged by electron emmission. If probe is negative wrt plasma, electrons can escape so probe current is decreased compared to what it would have been without electron emmission. Thus a probe characteristic (I-V curve) with probe hot and cold will differ if $V < V_p$ (the plasma potential) but not if $V > V_p$. Hence we can get at $V_p$. There is a sharp change in the probe current as the probe potential passes through the plasma potential.

## To calculate a probe characteristic

We want to know what the current to a probe inserted into the plasma, or a probe circuit, will be when we apply a voltage to the probe, either with respect to some defined ground plane such as the vacuum vessel, or with respect to another probe. We need to calculate what ions and electrons will arrive at the probe, considering potential effects.

Orbital theory - a simple picture (energy and momentum conservation) of charged particles which can hit a given surface. Tells us that under normal conditions (of ion attraction and electron repulsion) the electron current is the random drift current at a sheath edge times the Boltzmann exponential, and the ion current has saturated.

Collisionless sheath - tells us, by solving the energy, continuity and Poisson equations for the ions, that

a) ions must enter the sheath surrounding the probe at sound speed

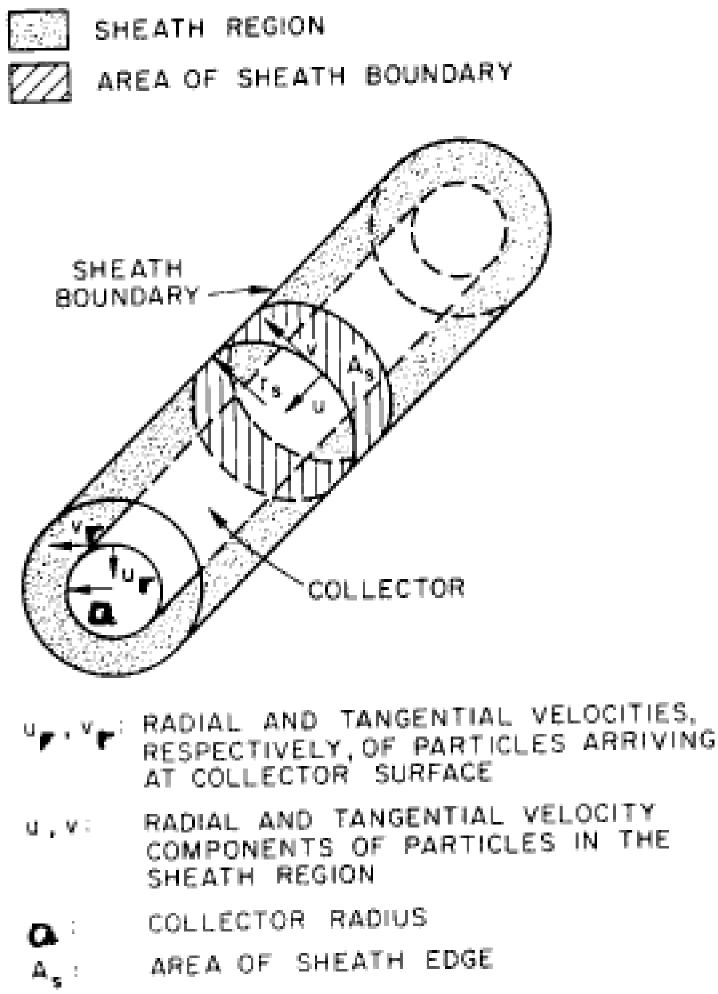
b) there must be a pre sheath to accelerate the ions to the sound speed

c) the total ion current at the sheath edge

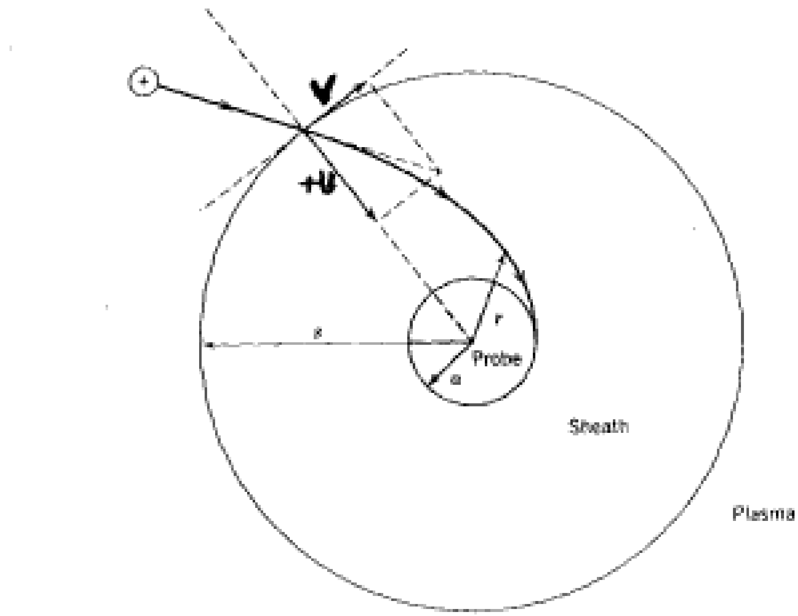
We now have an expression for the total (ion plus electron) current at the sheath edge, which allows us to calculate the characteristic (the I/V curve), and floating potential (probe potential when no current is drawn). For the double probe, write everything in terms of the ion saturation current, as it is better defined.

## Orbit theory (Orbital motion)

Consider a cylindrical wire of length l and radius r, $l >> r$, in a plasma, with a voltage applied. What will the current look like? In saturation conditions, where only one species is collected, we might expect the Child law to apply. But the collisionless trajectories prevent this from happening.




Consider a wire with negative potential with respect to the region around it. Then the wire repels negative ions and electrons, but attracts positive ions. Then it must be surrounded by a sheath or region of positive charge. This sheath is of such a size that the total positive charge in it equals the negative charge in the wire, so that the field of the wire does not extend beyond the sheath. The current cannot then exceed the rate at which ions arrive at the sheath edge. Suppose the negative potential to be large wrt the voltage equivalent of the ion velocities. Then the sheath can be considered in two regions. In the center is a region where most of the potential drop occurs, so that in this region only positive ions are found (and maybe a few electrons). Outside this region both positive and negative particles are found in approximately equal quantities, but conditions are modified because the ions are drawn to the collector. In this outer region the potential approaches the space value asymptotically. It is convenient to take as the sheath boundary the surface where the potential begins to drop sharply. This is O.K. because in the outer region the drop is small compared to the total drop. We regard the distribution of ions and electrons as known at the sheath boundary.




Note for figure: 'r' should be probe radius and 'a' should be sheath radius.

Assume end effects are negligible. There will be a circular sheath set up concentric with the cylinder, with radius a. Consider ions of one sign only (could be electrons). Let there be n per unit volume in a small element dτ bordering on the sheath. In a plane normal to the axis let u be the radial component of velocity and v the tangential component of velocity. u is positive if towards the wire. Then the number of ions in dτ with velocity components between u and u+du, v and v+dv, is

$$nf(u,\mathrm{v})du d\mathrm{v}$$

The total number of ions per unit length crossing the sheath edge with velocities within the given range is

$$2\pi anuf(u,\mathrm{v})dud\mathrm{v} \qquad 1)$$

Let $u_r$, $v_r$ be the velocities (radial and tangential components) of the ions arriving at the surface, and V the applied potential with respect to the sheath edge, **positive when the collector attracts ions.** Then for charge e and mass m, conservation of energy and angular momentum gives

$$\frac{1}{2}m\left(u_r^2+\mathrm{v}_\mathrm{r}^2\right)=\frac{1}{2}m\left(u^2+\mathrm{v}^2\right)+eV$$

$$r\mathrm{v}_r = a\mathrm{v}$$

These have a solution, for $u_r$ and $v_r$,

$$u_r^2 = u^2 - \left(\frac{a^2}{r^2} - 1\right)\mathrm{v}^2 + 2\frac{e}{m}V$$

$$\mathrm{v}_r = \frac{a}{r}\mathrm{v}$$

The ions which reach the surface of the collector must have

$$u > 0, \quad u_r^2 > 0$$

(Use squared to avoid imaginary solutions)

Now plot u, v as rectangular coordinates of a point the curve, then the curve $u_r^2 = 0$ is shown for Figure 1) V > 0 and Figure 2) V < 0.

For V > 0 (ion attraction) the value of v for u = 0 is

$$\mathrm{v} = \frac{2eV/m}{\left(a^2/r^2 - 1\right)}$$

The region where $u > 0, \quad u_r^2 > 0$ (i.e. where particles will be collected) is valid is thus between the solid lines, and to the right of the v axis. The total no. of particles per unit length reaching the collector per second is found by integrating equation 1 over the region between the solid lines. For a given value of u, v must lie between $-\mathrm{v}_1$ and $\mathrm{v}_1$, where these are found by solving the equation for $u_r^2 = 0$

$$\mathrm{v}_1^2 = \frac{r^2}{a^2 - r^2}\left(u^2 + 2\frac{e}{m}V\right)$$

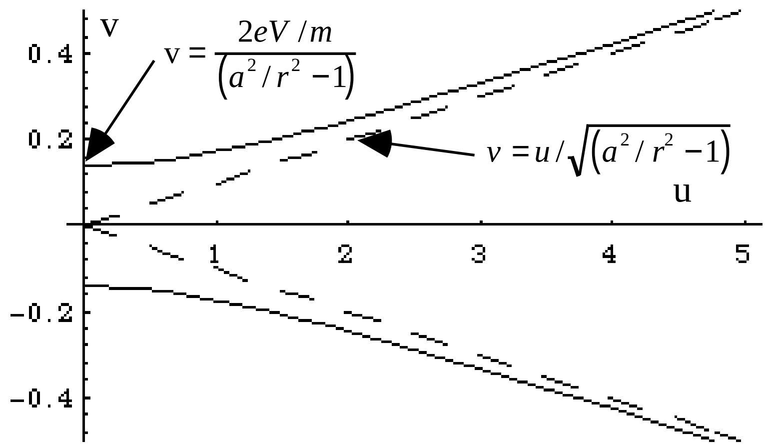



Figure 1. Solid lines; the curve of v against u for V positive (collection). The dashed lines correspond to the curve $v = u/\sqrt{\left(a^2/r^2 - 1\right)}$. r = 1, a = 10, V = 1, m = 1, e = 1

For V < 0 (Figure 2) the region where $u > 0, \quad u_r^2 > 0$ is again within the solid lines. Note that u cannot be less than a certain value, found when v = 0:

$$u_1^2 = -2\frac{e}{m}V$$

For any u > $u_1$ the values of v lies between -$v_1$ and $v_1$, as defined previously.

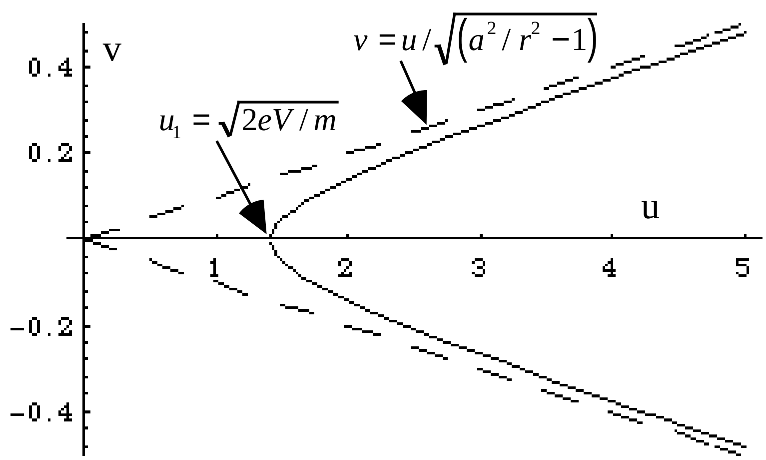



Figure 2. Solid lines; the curve of v against u for V negative (repulsion). The dashed lines corresponds to the curve $v = u/\sqrt{\left(a^2/r^2 - 1\right)}$. r = 1, a = 10, V = 1, m = 1, e = 1

Now we can write an expression for the total current $I_e$ of the one species under consideration to the collector. It is e times l time expression 1, integrated suitably

$$I_e = 2\pi a \ln e \int_{0,u_1}^{\infty} \int_{-\mathrm{v}_1}^{\mathrm{v}_1} u f(u,\mathrm{v}) d\mathrm{v} du$$

where the lower limit of u is either 0 for V > 0, or $u_1$ for V < 0. We can replace n by an equivalent expression involving the total current $i_e$ crossing a unit area at the sheath edge (the drift current of the species e). This is given by

$$i_e = ne \int_0^{\infty} \int_{-\infty}^{\infty} u f(u,\mathrm{v}) d\mathrm{v} du \qquad 9)$$

## The Distribution Function

For a 1 D Maxwellian we derive:

$$f(u) = A e^{\left(-\frac{1}{2} m u^2 / (kT)\right)}$$

$$A = n \left(\frac{m}{2\pi kT}\right)^{1/2}$$

For a 3 D Maxwellian we derive

$$f(u,v,w) = A_3 e^{\left(-\frac{1}{2} m\left(u^2 + v^2 + w^2\right)/(kT)\right)}$$

$$A_3 = n \left(\frac{m}{2\pi kT}\right)^{3/2}$$

By analogy, for a 2 D Maxwellian we have

$$f(u,\mathrm{v}) = A_2 e^{\left(-\frac{1}{2} m\left(u^2 + \mathrm{v}^2\right)/(kT)\right)}$$

$$A_2 = n \left(\frac{m}{2\pi kT}\right)$$

Note our definition in equation 0 and 1 means that n = 1 (the number density n or N appears in the definition)

Substitute into 9 gives

$$i_e = ne \left(\frac{m}{2\pi kT}\right) \int_0^{\infty} \int_{-\infty}^{\infty} u e^{\left(-\frac{1}{2} m\left(u^2 + \mathrm{v}^2\right)/(kT)\right)} d\mathrm{v} du$$

Integrating over v gives

$$i_e = ne\left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}}\int_0^\infty ue^{\left(-\frac{1}{2}m\left(u^2\right)/(kT)\right)}du$$

(note $\int e^{-x^2}dx = \frac{\sqrt{\pi}Erf(x)}{2}$; $Erf(x=\infty)=1$; $Erf(x=-\infty)=-1$)

Then integrating over u gives

$$i_e = ne\left(\frac{kT}{2\pi m}\right)^{\frac{1}{2}}$$

This is the formula of the kinetic theory for the drift current of e.g. the ions.

Now we must evaluate the current to the cylindrical collector $I_e$. Make the substitution

$$\eta = \frac{eV}{kT}$$

and use the variables of integration

$$x = u\sqrt{\frac{m}{2kT}}; \quad y = \mathrm{v}\sqrt{\frac{m}{2kT}}$$

Then write the current to the probe $I_e$ in terms of the drift current $i_e$:

$$I_e = 2\pi a \ln e\int_{0,u_1}^{\infty}\int_{-\mathrm{v}_1}^{\mathrm{v}_1} uf(u,\mathrm{v})d\mathrm{v}du = 8\sqrt{\pi}ali_e\int_{0,\sqrt{-\eta}}^{\infty}\int_0^{r\sqrt{x^2+\eta}/\sqrt{a^2-r^2}} xe^{-\left(x^2+y^2\right)}dydx$$

Integrate by parts using Mathematica. For $\eta > 0$ (attraction) we get

$$I_e = 2\pi rli_e\left[\frac{a}{r}Erf\left(\sqrt{\frac{\eta r^2}{a^2-r^2}}\right)+e^{\eta}\left(1-Erf\left(\sqrt{\frac{\eta a^2}{a^2-r^2}}\right)\right)\right]$$

This saturates at large $\eta$ (i.e. large attractive applied voltage) at

$$I_e = 2\pi ali_e$$

For $\eta < 0$ (repulsive) we get

$$I_e = 2\pi rli_e e^{\eta}$$

Drop subscript e (distinguishing the fact that we are only considering one charge species at a time) mostly from here on. For retarding potentials ($\eta < 0$) the current is independent of the sheath radius a, and its logarithm is a linear function of the collector voltage. The slope gives the temperature T. Knowing the size of the probe (i.e. r and l) we then get the density of the plasma n

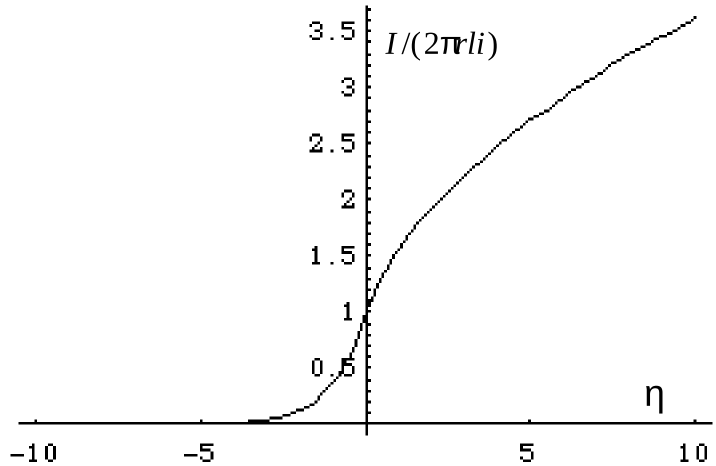
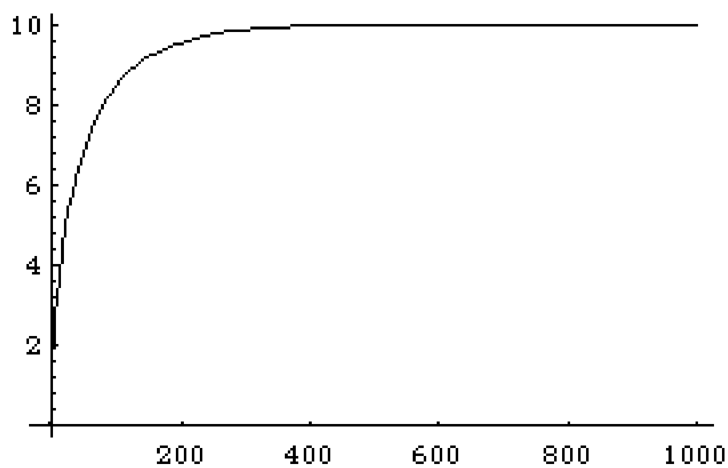



Figure 3. The normalized current ($I/(2\pi rli)$) - voltage ($\eta = eV/(kT)$) characteristic for a cylindrical probe, with r = 1, a = 10, m = 1, e = 1. The saturation value for large $\eta$ is 10.




As above, but extended to large $\eta$ to show saturation.

In the region $\eta > 0$, if we assume large a/r, we can approximate the expression for the current by

$$I = 2\pi rli \frac{2}{\sqrt{\pi}} \sqrt{1+\eta}$$

This is shown, together with the exact expression, in Figure 4.

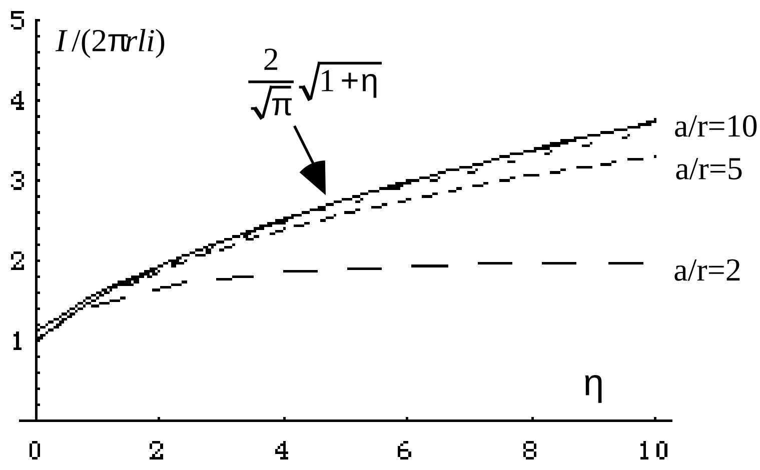



Figure 4. The normalized current ($I/(2\pi rli)$) - voltage ($\eta = eV/(kT)$) characteristic for a cylindrical probe, with a/r = 2, 5 and 10, and the approximate expression

$$I = 2\pi rli \frac{2}{\sqrt{\pi}}\sqrt{1+\eta}$$

i.e. we have

$$\left(\frac{I}{2\pi rli}\right)^2 = \frac{4}{\pi}\left(1+\frac{eV}{kT}\right)$$

Therefore plot the square of the current per unit area against the applied voltage, and obtain a straight line. The intercept of this line on the voltage axis line gives

$$V_1 = \frac{-kT}{e}$$

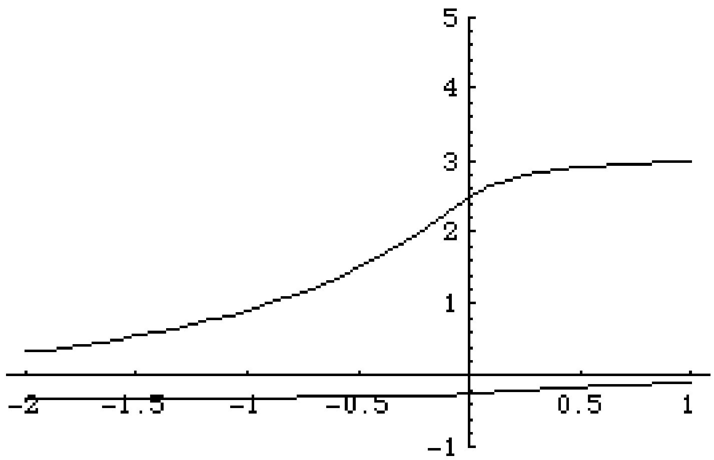
i.e. we can deduce the origin of potentials, the space potential.

The slope is

$$S = \frac{4}{\pi}\frac{e}{kT}i^2 = \frac{2}{\pi^2}\frac{e}{m}(ne)^2$$

i.e. hence n

Now what happens to a cylindrical electrode as we change voltage in a plasma with electrons and ions present? Remember that our potential is with respect to the plasma potential, the potential at the sheath edge. We have not asked what this is. There is a floating potential when the net current, ion pus electron, is zero. This is the situation before any applied potential is switched onto the probe. It is the potential that the probe would sit at with no applied voltage.




Electron (positive) and Ion (negative) currents collected to a single probe as a function of applied voltage, for $m_i/m_e = 100$, all other parameters = 1.

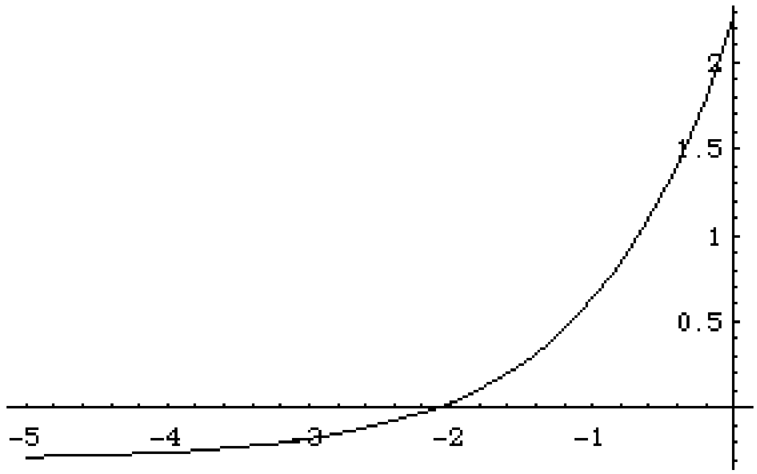
All things being equal except the electron and ion masses, the electron current is much bigger than the ion current, (m is at least 1840 times smaller). Therefore this floating potential must occur when the electrons are well into the retarding regime ($\eta < 0$), to get the electron current down to the ion current value. In this situation the ions will be well into the attracting regime ($\eta > 0$), i.e. the ion current will be in the saturated state. Now we apply perturbations about this. Negative voltages attract ions, and positive voltages attract electrons. Making the voltage more negative than the floating potential keeps the ions in saturation, but the electron current becomes even smaller. Making the applied potential positive will start to change the ion current, but in a negligible amount to the electron current, which changes much faster and is much bigger. Therefore we can approximate the situation by assuming that the ion current is always in saturation (when it is not, as V is increased positive, the error is small), and keeping the electrons in the retarding situation. This is so as long as the applied voltage is not much large than the floating potential. Then the 'characteristic' becomes

$$I_{probe} = I_{ion-saturation}\left(1 - e^{e\left(V - V_{float}\right)/kT}\right)$$

where

$$I_{ion-saturation} = 2\pi r\, l n e \left(\frac{kT}{2\pi m}\right)^{\frac{1}{2}}$$

W e derive this more accurately later on.




Summed electron (positive) and ion (negative) currents as a function of applied voltage.

## Langmuir floating probes

Probe models developed by Tonks and Langmuir in the 20's. Still being used, and still not understood completely. Used to measure density, temperature, potential, both equilibrium and fluctuations. For: easy to use, Against: low temperatures, effects of B fields, non-Maxwellian distributions,

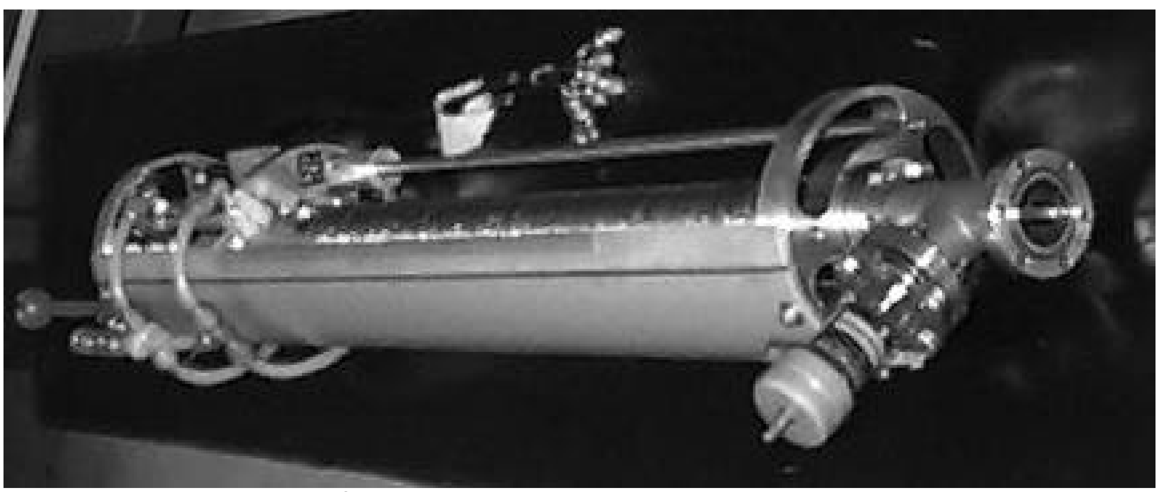


Figure 2. A photograph of an FRC Langmuir probe head which is attached to a reciprocating drive. The probe head is at the far right hand side.

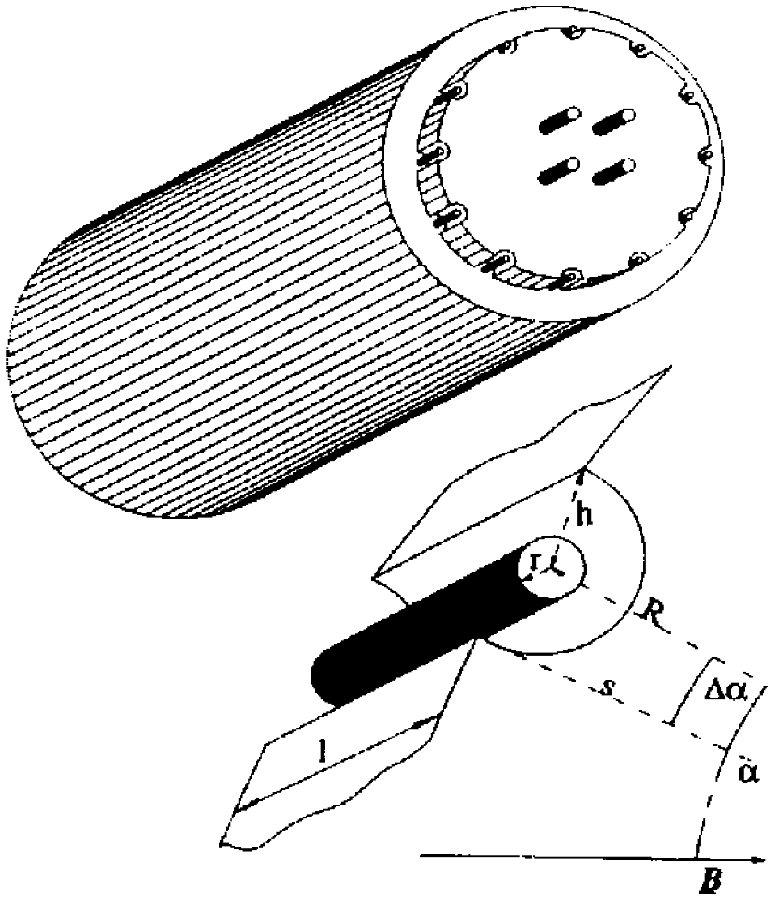
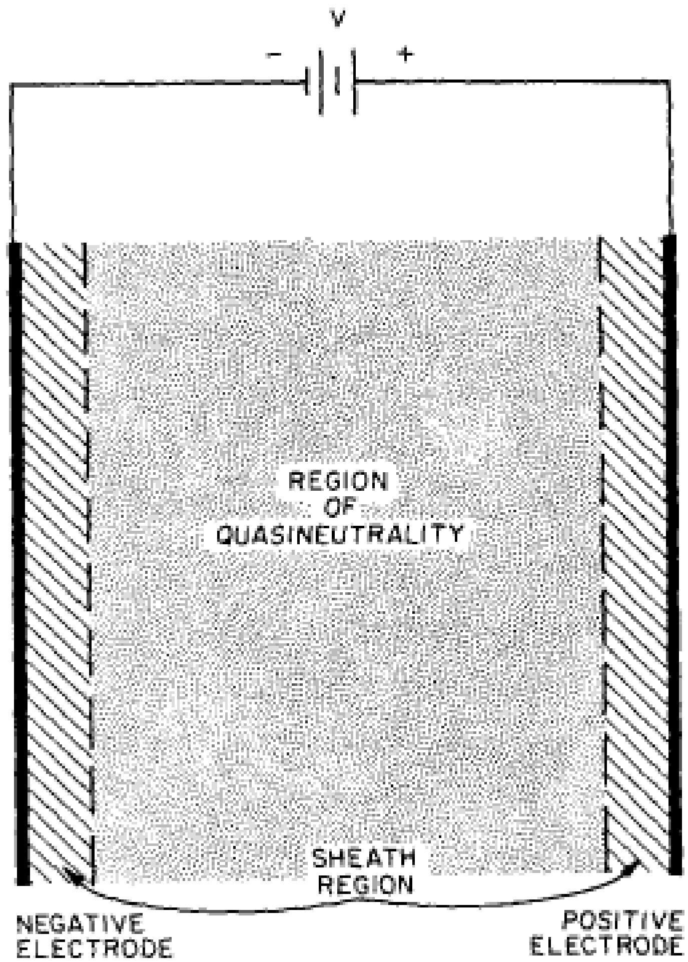



Figure 3. The geometry of the combined multi-pin head used for density, temperature, potential and velocity measurements. The lower figure is a detail of one of the twelve outer pins used for the velocity measurements.

What happens when a conductor is placed in a plasma? Consider a two electrode system. Potential applied between two electrodes, ions (positive) attracted towards negative electrode, electrons toward positive electrode. Charge separation caused by applied electric field opposed by electric fields created by displaced electrons and ions. Sheath set up in region of surfaces.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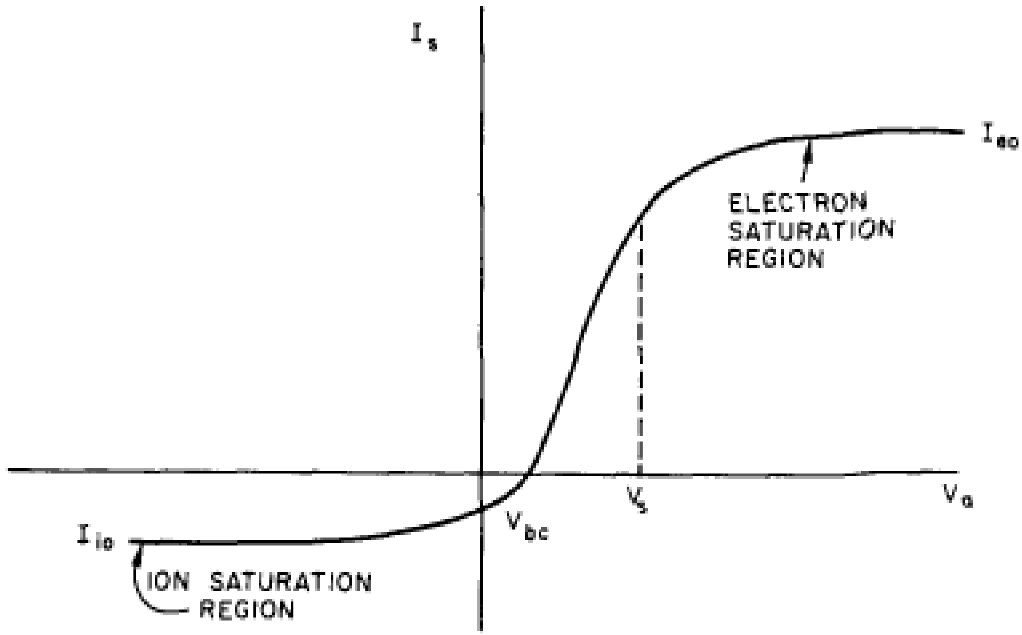



V applied to a single probe, then current is collected - see figure above and previous section, the characteristic. At some V = $V_s$ the probe and plasma potential are equal. Then no electric field, and electrons and ions approach probe with random thermal velocities, given by

$$v_{th} = \sqrt{\frac{8kT}{\pi m}}$$

Electrons travel faster than ions. Outside sheath $n_e = n_i$ (neutral plasma). Thus when probe is at $V_s$, more electrons hit the probe than ions (per unit time). Thus current collected is largely an electron current $I_{e0}$. Increase applied V, making probe more positive wrt plasma. Then the electrons are accelerated and the small number of ions are repelled. Since current collected is due to electrons entering sheath with random thermal motions, and since sheath size about constant, then current does not change much. This is called electron saturation. Now decrease V below Vs. Now electrons are repelled and ions accelerated. At V = $V_{bc}$ then currents are equal: the floating potential. Further reduce V then electrons repelled and only ions are collected: ion saturation.

See below for a two electrode system. If potential is positive the current (ion plus electron) between probes 1 and 2 is defined to be positive. Current flowing in circuit is $I_s$, and plotted against $V_a$ in a figure below.

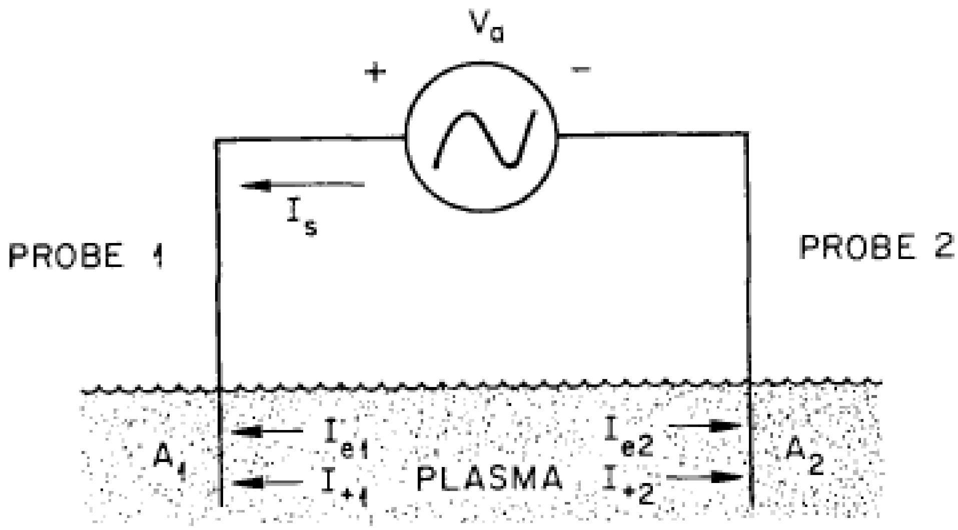



Fig. 3. Schematic diagram of a floating-probe system inserted into a macroscopically neutral plasma. Probes 1 and 2 have associated areas $A_1$ and $A_2$, respectively. The current $I_s$ is defined to be positive in the direction shown.

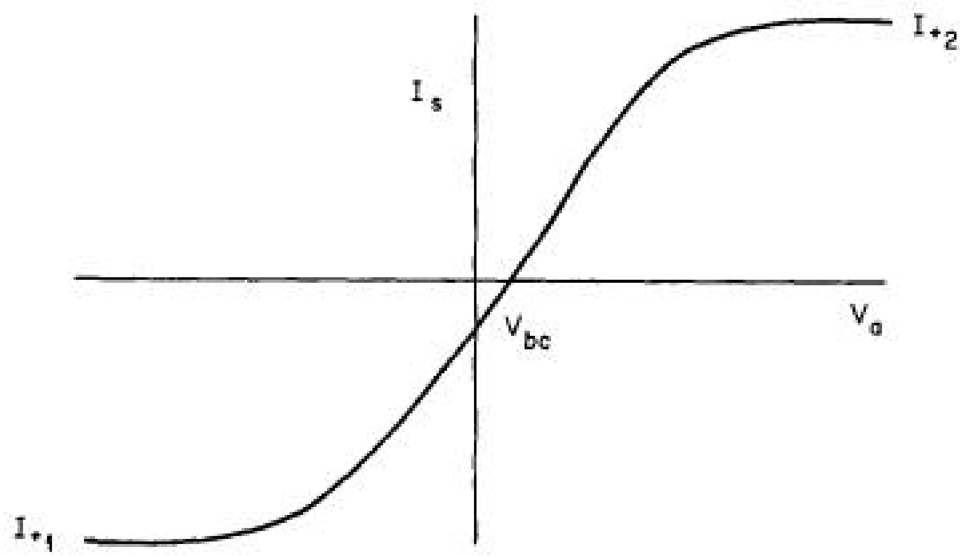



Fig. 4. Ideal double-probe characteristic.

If potential of probe 2 << that of probe 1, then current collected by probe 2 is about equal to ion sat current $I_{i2}$ Probe 2 repels almost all electrons, so total current in system is dominated by ion sat current. As V is deceased, making p2 less negative wrt p1, current collected remains about constant until some electrons are collected by p2. Decease V more then more electrons collected by p2 and current decreases. $V_{bc}$ is point where electron and ion currents are zero and no net current is drawn. As p1 is made negative wrt p2 then situation reverses. If two probes have equal area then characteristic symmetric about $I_s = 0$, and $I_{i1} = I_{i2}$.

Potential distribution shown below.

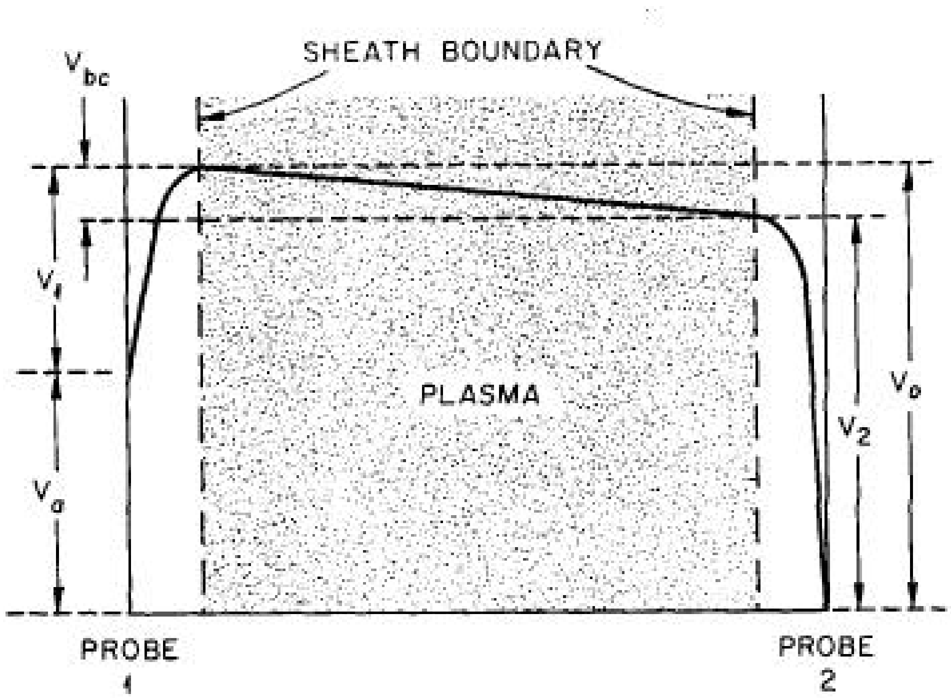



Fig. 5. Potential distribution between the two electrodes of a floating double-probe system.

Usually the electrodes are negative wrt the surrounding plasma. The pot drop $V_{bc}$ is normally the result of spatial variations rather than the pot drop across the ohmic resistance of the plasma. Therefore $V_{bc}$ remains independent of the current flowing. $V_{bc}$ therefore is independent of the applied potential $V_a$.

Potential difference between tips is given by

$$V = V_1 - V_2 = V_{bc} - V_a ,$$

i.e.

$$V_a + V_1 = V_{bc} + V_2$$

and current continuity gives

$$I_s = I_{e1} - I_{i1} = I_{i2} - I_{e2}\text{, i.e. } I_{e1} + I_{e2} = I_{i2} + I_{i1}$$

Now, what to use for the currents? Remember $I = 2\pi r l i e^{\eta}$ for the case of interest, (retarding potentials) where $\eta = eV/(kT)$, which in our present terminology, with $V_e = kT_e$ becomes

$$I_e = 2\pi r l i_e e^{V/V_e}$$

where we are using $i_e$ for the random electron drift current. Then

$$I_{i1} + I_{i2} = \Sigma I_i = \Sigma I_e = i_{e1} 2\pi r_1 l_1 e^{V_1/V_e} + i_{e2} 2\pi r_2 l_2 e^{V_2/V_e}$$

Rearrange to give:

$$\Sigma I_i = I_{e2}\left[1 + \frac{i_{e1} 2\pi r_1 l}{i_{e2} 2\pi r_2 l_2} e^{\frac{V_1 - V_2}{V_e}}\right]$$

Now substitute the voltage relationship:

$$\Sigma I_i = I_{e2}\left[1 + \frac{i_{e1} 2\pi r_1 l}{i_{e2} 2\pi r_2 l_2} e^{\frac{V_{bc} - V_a}{V_e}}\right]$$

Equal areas, lengths, etc.

$$2I_i = I_{e2}\left[1 + e^{\frac{V_{bc} - V_a}{V_e}}\right]$$

Then

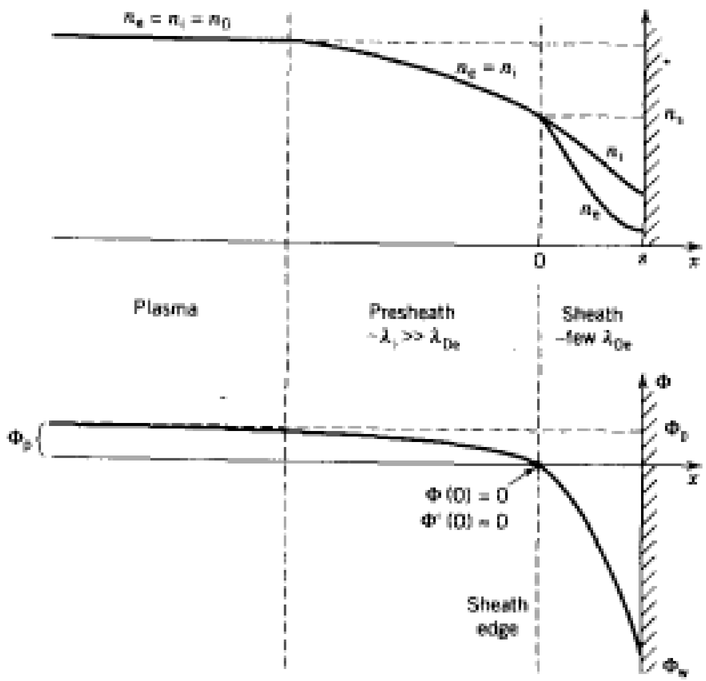
$$I_s = I_i - I_{e2} = I_i \left[\frac{e^{(V_{bc} - V_a)/V_e} - 1}{e^{(V_{bc} - V_a)/V_e} + 1}\right] = I_i \tanh\left[\left(V_{bc} - V_a\right)/(2V_e)\right]$$

I

So, measure I-V characteristic of a floating double probe and get $I_i$ and $T_e$. But what is $I_i$?

## The collisionless sheath.

What is the ion current? Assume Maxwellian electrons, cold ions, $n_e = n_i$ at the sheath plasma interface x = 0. Define potential $\varphi = 0$ at x = 0. Ion energy (directed) is $u_s$.




Ion energy conservation

$$\frac{1}{2}Mu^2(x) = \frac{1}{2}Mu_s^2 - e\varphi(x)$$

Continuity

$$n_i(x)u(x) = n_{is}u_s$$

Solve for u from 1 and substitute into 2

$$n_i = n_{is}\left(1 - \frac{2e\varphi}{Mu_s^2}\right)^{-1/2}$$

Electrons are Boltzmann

$$n_e(x) = n_{es}e^{\varphi(x)/kT_e}$$

Set $n_{es} = n_{is}$ at the sheath edge, and substitute $n_e$ and $n_i$ into Poisson's equation

$$\frac{d^2\varphi}{dx^2} = \frac{e}{\varepsilon_0}(n_e - n_i)$$

Then

$$\frac{d^2\varphi}{dx^2} = \frac{en_s}{\varepsilon_0}\left( e^{\varphi/T_e} - \left(1 - \frac{\varphi}{E_s}\right)^{-1/2} \right)$$

where $eE_s = \frac{1}{2} Mu_s^2$ is initial ion energy

We have a nice nonlinear equation for  you. It has stable solutions only under certain conditions (a given $u_s$, which means the ions have been accelerated)

## Bohm sheath criterion

Integrate by multiplying by d$\varphi$/dx and integrate over x

$$\int_o^\varphi \frac{d\varphi}{dx}\frac{d}{dx}\left(\frac{d\varphi}{dx}\right)dx = \frac{en_s}{\varepsilon_0}\int_0^\varphi \frac{d\varphi}{dx}\left( e^{\varphi/T_e} - \left(1 - \frac{\varphi}{E_s}\right)^{-1/2} \right)dx$$

cancel dx's and integrate wrt $\varphi$:

$$\frac{1}{2}\left(\frac{d\varphi}{dx}\right)^2 = \frac{en_s}{\varepsilon_0}\int_0^\varphi \left( T_e e^{\varphi/T_e} - T_e + 2E_s\left(1 - \frac{\varphi}{E_s}\right)^{1/2} - 2E_s \right)$$

where we have assumed $\varphi = 0$ and d$\varphi$/dx = 0 at x = 0 (not strictly true). Now RHS must be > 0 for a solution to exist (means $n_e < n_i$ in the sheath). Expand RHS in Taylor series and find

$$(f(x+h) = f(x) + hf'(x) + h^2/2f''(x))$$

$$\frac{1}{2}\frac{\varphi^2}{T_e} - \frac{1}{4}\frac{\varphi^2}{E_s} > 0$$

i.e. $E_s > T_e/2$ or $u_s > u_b = (eT_e/M)^{1/2}$. This **Bohm criterion** says that the ions must enter the sheath at the speed stated, and this requires some pre sheath to accelerate them.

## The pre sheath

The potential drop to accelerate the ion is

$$\frac{1}{2} Mu_b^2 = e\varphi_p$$

i.e.

$$\varphi_p = T_e / 2$$

The ratio of the density at the sheath $n_s$ to that in the bulk plasma where the pre sheath ends $n_b$ is

$$n_s = n_b e^{-\varphi_p T_e} = 0.61 n_b$$

## Floating potential

The full current to a conducting surface can now be approximated. The electron current is

$$I_- = Ane\left(\frac{kT_e}{2\pi m_e}\right)^{\frac{1}{2}} e^{V/T_e} = Ane\left(\frac{T_e}{m_i}\right)^{1/2} \frac{1}{2}\left(\frac{2m_i}{\pi m_e}\right)^{1/2} e^{V/T_e}$$

Ion current is

$$I_+ = -Ane\left(\frac{kT_e}{m_e}\right)^{\frac{1}{2}} e^{-1/2}$$

where the last factor accounts for the fact that the sheath density is less than the density at infinity. Then when total current = 0 we get the floating condition

$$\frac{eV_f}{T_e} = \frac{1}{2}\left[\ln\left(\frac{2\pi m_e}{m_i}\right) - 1\right]$$

thus measuring the floating potential gives the temperature directly.

# The Triple Probe (and variants)

There are two open circuit probes or floating probes (2, 3), whose potential is measured wrt ground. There is a strongly negatively biased probe for which no electrons are collected, and which measures the ion saturation current. The potential of the return probe (probe 4) is also measured.

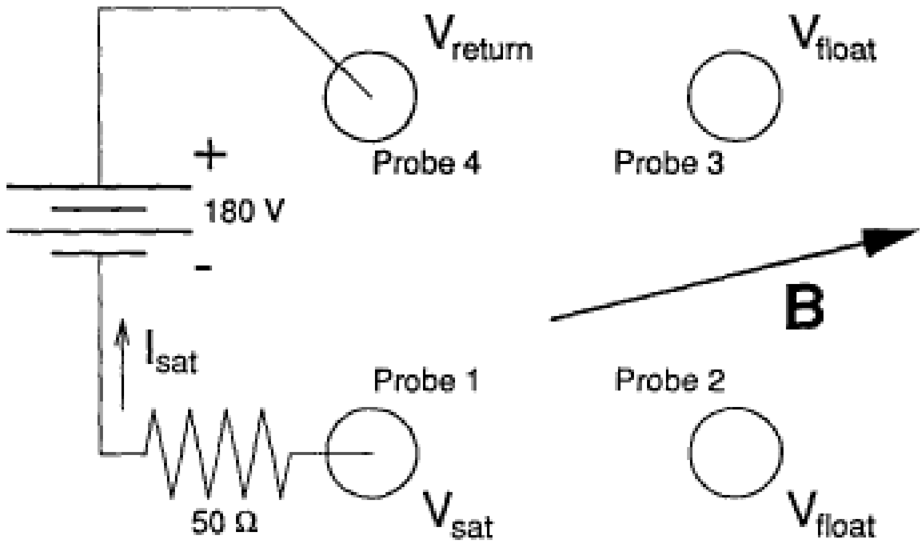
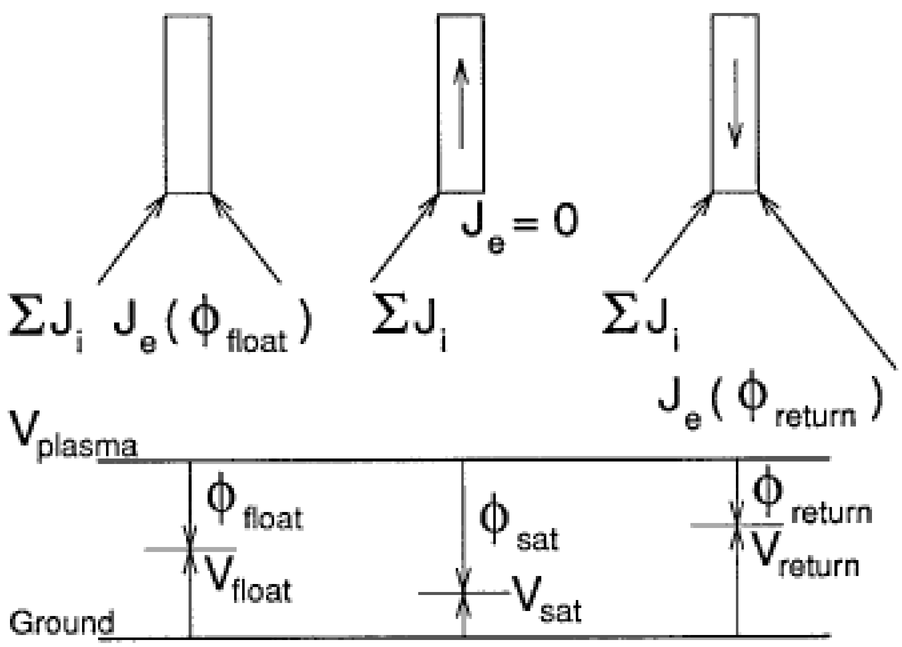



Figure 3.3: Arrangement of the FRLP Array.




Probes 1, 4 and 3 constitute the basic triple probe configuration. A battery is used to bias probe 1 to 180 V below probe 4. Probes 4 and 3 are used to calculate $T_e$. Probe 3 is used to compute the plasma potential . The current collected by probe 1 is passed through a 50 Ohm resistor and returned to the plasma (a double probe). Measuring the voltage across the resistor gives the saturation current, from which the density is calculated. Probe 2 is not a part of the triple probe set up, but provides with probe 3 information on fluctuations.

probe 2 and 3: floating so $I_e + I_i = 0$. The ion current is the saturation current

probe 1: collects $I_i$, while $I_e = 0$. $I_i$ is the saturation current.

probe 4: must collect $2I_e$ measured at $\varphi_{float}$ so that net current with probe 1 is zero: potential of probe must adjust to this situation

i.e.

$$I_e(\varphi_{float}) = \frac{1}{2} I_e(\varphi_{return})$$

$$Ane\left(\frac{kT_e}{2\pi m_e}\right)^{\frac{1}{2}} e^{\varphi_{float}/T_e} = \frac{1}{2} Ane\left(\frac{kT_e}{2\pi m_e}\right)^{\frac{1}{2}} e^{\varphi_{freturn}/T_e}$$

$$T_e = e\left[\frac{\varphi_{return} - \varphi_{float}}{\ln(2)}\right]$$

hence $T_e$.

Ion saturation current (probe 1):

$$I_+ = -Ane\left(\frac{kT_e}{m_e}\right)^{\frac{1}{2}} e^{-1/2},$$

hence, with $T_e$, have $n_e$.

Floating potential (probes 2,3):

$$\frac{eV_f}{T_e} = \frac{1}{2}\left[\ln\left(\frac{2\pi m_e}{m_i}\right) - 1\right]$$

remember this is wrt $\varphi_p$!!!. Then $\varphi_p = V_{float} - \frac{T_e}{2e}\left[\ln\left(\frac{2\pi m_e}{m_i}\right) - 1\right]$

## Effects of B field.

Use modified surface area.

$$i = 2\pi a l N e \int_{0,u_1}^{\infty} \int_{-v_1}^{v_1} u f(u,v)\, dv\, du$$

$$i_{sat} = \lim_{a \to \infty} 2\pi a l N e \int_{0,u_1}^{\infty} u \int_{-v_1}^{v_1} u f(u,v)\, dv\, du$$

$$v_1^2 = \left(\frac{r^2}{a^2 - r^2}\right)\left(u^2 + 2\frac{e}{m}V\right)$$

replace $f(u,v)\, dv$ with $\frac{dF(u,v)}{dv} \cdot dv$

$$\rightarrow i_{sat} = \lim_{a \to \infty} 2\pi a l N e \int_{0,u_1}^{\infty} u \left[F(v_1) - F(-v_1)\right] du = f(a)\, g(a)$$

$g(a) = \left(\frac{1}{a}\right)$

note $v_1 \to 0 \to F(v_1) - F(-v_1) \to 0$, and $a \to \infty \to 0 \cdot \infty$

$f(a) = u$

$g(a) = \int F(v_1) du$

$\frac{dg(a)}{da} = \frac{-1}{a^2}$ $\quad \frac{dF}{da} =$

$$2\pi l N e \int_{0,u_1}^{\infty} u \cdot \left[\frac{dF(v_1)}{dv_1}\frac{dv_1}{da} - \frac{F(-v_1)}{dv_1}\frac{dv_1}{da}\right] du$$

now note $\frac{dF(v_1)}{dv_1} \equiv -\frac{dF(-v_1)}{dv_1}$

$$i_{sat} = \lim_{a \to \infty} \frac{f'(a)}{g'(a)} = \lim_{a \to \infty} 2\pi (-a^2) l N e \int_{0,u_1}^{\infty} 2u \frac{dF(v_1)}{dv_1} \frac{dv_1}{da}\, du$$

$$= \lim_{a \to \infty} 2\pi(-a^2) l N e \int_{0,u_1}^{\infty} 2u f(u,v_1) \frac{dv_1}{da}\, du$$

$$= \lim_{a \to \infty} 2\pi a^2 l N e \int_{0,u_1}^{\infty} 2u f(u,v_1) \cdot \frac{-ra}{(a^2 - r^2)^{3/2}} \sqrt{u^2 + \frac{2eV}{m}}\, du$$

$$= 4\pi r l N e \int_{0,u_1}^{\infty} u f(u,0) \cdot \sqrt{u^2 + \frac{2eV}{m}}\, du$$